

Optimal Stabilization and Dynamic Programming

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Abstract

In this article, we present and discuss the infinite horizon problem of optimal stabilization. Besides, the optimality conditions in the form of an Hamilton-Jacobi-Bellman equation, we present also a method to define a feedback control strategy.

1 INTRODUCTION

In this article, we present and discuss the infinite horizon problem of optimal stabilization. Besides, the optimality conditions in the form of an Hamilton-Jacobi-Bellman equation, we also present a method to synthesize a feedback control strategy.

In many references, by optimal stabilization it is meant time-optimal stabilization, i.e. finding a control that steers a system to the origin in minimum time. However, here, our goal is substantially different.

Given a dynamic control system and a set in the state space containing at least one equilibrium point, we are interested in finding a feedback control strategy which stabilizes the system in the sense that the corresponding trajectory converges asymptotically to an equilibrium point and, simultaneously, minimizes a given cost functional.

Notice that the value of the optimal cost depends not only on the equilibrium point, but also on the particular trajectory driving the system to this point.

The literature reveals a vast demand for the type of result that we consider in this article. A small sample of practical optimal stabilization problems include micro-electro-mechanical (MEMS) control systems [4], economic systems under a variety of constraints and assumptions, [3, 10, 16], rigid body mechanical systems [8], biological, medical, health care systems [9], and general chaotic systems [1], to name just a few.

This contrasts strikingly with the what seems to be the small body of results in the general nonlinear dynamic optimization framework addressing the pertinent issues. See for example [17] for results on the stabilization and minimax optimal control in

the context of stochastic control systems, and [11] for a very specific problem and approach.

The problem of stabilizing general nonlinear dynamic control systems has been receiving a considerable attention in the control literature, [13, 14, 6, 15] and references cited therein. It has also emerged the important role of dynamic optimization results and techniques and methods of nonsmooth analysis to derive stability results, see [6, 5].

However, to the best of our knowledge no results have been derived for optimal control problems where control strategies are restricted to the subset of stabilizing ones.

This article is organized as follows. In the next section, we introduce a precise and detailed statement of our problem. Then, in section three, the Hamilton-Jacobi-Bellman equation corresponding to this problem is presented followed in the ensuing section by the definition and pertinent results concerning the verification function. In section five, a mechanism of feedback synthesis based on a dynamic programming approach is presented and a result on the convergence of a sequence of sampled feedback control processes is discussed and proved. Finally, some brief conclusions are presented.

2 PROBLEM STATEMENT

Let us consider the following dynamic control system

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)), & t \in [\tau, +\infty) \text{ a.e.}, \\ x(\tau) = z, \end{cases} \quad (1)$$

where

$$u \in \mathcal{U} := \{u(\cdot) \in L^\infty[\tau, \infty) : u(t) \in \Omega \subset \mathbb{R}^m \text{ a.e.}\}, \quad (2)$$

with Ω being a closed convex set. Assume that the closed set $S \subset \mathbb{R}^n$, called target set, contains at least an asymptotic equilibrium of the former, i.e., $\exists \xi \in S$ and $\bar{u}(\cdot) \in \mathcal{U}$ such that $x_{\bar{u}}(\cdot)$, the response of the system to the control \bar{u} with $x_{\bar{u}}(\tau) = z$, satisfies

$$x_{\bar{u}}(t) \rightarrow \xi \text{ as } t \rightarrow \infty.$$

We consider the following optimal control problem:

$$\begin{aligned} P_\infty(\tau, z) \quad & \text{Minimize } g(\xi) + \int_\tau^\infty e^{-\delta t} f_0(x(t), u(t)) dt \quad (3) \\ & \text{subject to } (1), (2), \text{ and} \\ & x(t) \rightarrow \xi \text{ as } t \rightarrow \infty, \quad (4) \\ & \xi \in S \subset \mathbb{R}^n \quad (5) \end{aligned}$$

Here, the constant $\delta > 0$ is the discount rate, $f_0 : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}$ and $f : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ are given functions, and $S \subset \mathbb{R}^n$ is a closed set.

This is an infinite time horizon problem in which the optimization is taken over arcs x such that $x(t)$ converges asymptotically to an equilibrium point ξ of (1), and the ξ is also a choice variable. The pair (x, ξ) satisfying the constraints (1), (2), (4), and (5) is called an admissible process of $P_\infty(\tau, z)$. We sometimes refer to an admissible arc x leaving implicit the existence of $\xi \in S$ such that the pair (x, ξ) is an admissible process of $P_\infty(\tau, z)$.

We now clarify the sense of the convergence $x(t) \rightarrow \xi$ as t goes to ∞ . We remind the reader that the optimal trajectory must reach or approach S , but may not stop in the first point in S . It can happen that to minimize g over points in S which are limits of admissible trajectories, the optimal trajectory must reach S and stays there for a long time before approaching the optimal $\xi \in S$.

To replace the convergence restriction with a more tractable condition we found convenient to introduce the following.

We say that $x(t) \rightarrow \xi \in S$, as $t \rightarrow \infty$, for some $\bar{x} \in \mathbb{R}^n$, if for some $\gamma > 0$, $\int_\tau^t e^{\gamma s} \|x(s) - \xi\| ds$ is finite for all $t \in [\tau, \infty)$.

We point out that the choice of γ is dependent on the specific application being considered.

Observe that we have here the problem of minimizing some cost function restricted to admissible arcs which are convergent to a critical point of f . This fact can be regarded as restriction defined as above. Standard theories for optimal control are not capable of dealing with this type of constraint directly.

We propose to include this restriction as a penalized cost added to the cost function in the original problem. The reason is that we only need that the integral be finite. So, we substitute problem $(P_\infty(\tau, z))$ by a l -parameterized problem posed as follows:

$$\begin{aligned} & \text{Minimize} \\ & g(\xi) + \int_\tau^\infty e^{-\delta t} f_0(x(t), u(t)) dt + \int_{\tau+l}^\infty e^{\gamma s} \|x(s) - \xi\| ds \quad (6) \\ & \text{subject to } (1), (2), \text{ and} \\ & \xi \in S \subset \mathbb{R}^n. \quad (7) \end{aligned}$$

Note that when $l \rightarrow \infty$ we expect that

$$\int_{\tau+l}^\infty e^{\gamma s} \|x(s) - \xi\| ds$$

will go to zero, thus recovering the original optimization problem without the explicit restriction.

Having done that, we show how to construct an (almost) optimal feedback control for problem $P_\infty(\tau, \xi)$. The proposed framework allows us to construct feedback controls which stabilizes the system of problem $P_\infty(\tau, \xi)$ while optimizing it.

Let us now set up our basic assumptions.

Let $F : [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R} \times \mathbb{R}^{n+1}$ be defined by

$$F(t, x, u) = \text{col}\{e^{-\delta t} f_0(x, u), f(x, u)\}.$$

We will require the data of our problem to satisfy the following assumptions:

- H1) F is a continuous and $F(t, \cdot, u)$ is locally Lipschitz;
- H2) There exists $c > 0$ such that
$$F(t, x, u) \in c(1 + \|x\|)B \quad \forall (t, x) \in [0, \infty) \times \mathbb{R}^n;$$
- H3) $\forall (t, x) \in [0, \infty) \times \mathbb{R}^n$ the set $F(t, x, \Omega)$ is convex valued;
- H4) The set Ω is compact; and
- H5) g is lower semicontinuous.

3 Hamilton-Jacobi-Bellman Equation

In this work we will propose to construct an optimal solution of optimal stabilization problem by using the concept of verification function. It happens that verification functions, which will be defined in the next section, can be shown to be solutions of the so called Hamilton-Jacobi-Bellman (HJB) partial differential equation related to the control problem.

Let $\mathcal{H} : [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ denote the Hamiltonian function defined, for this problem by

$$\mathcal{H}(t, x, \eta) := \sup_{v \in \Omega} \{ \langle \eta, f(x, v) \rangle + e^{-\delta t} f_0(x, v) \}. \quad (8)$$

Then, the continuous function $\phi : [\tau, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R}$ is said to be the well known viscosity solution to the HJB equation if

$$\phi_t(t, x) - \mathcal{H}(t, x, -\phi_x) = 0 \quad \forall (t, x) \in [\tau, \infty) \times \mathbb{R}^n,$$

if

$$\nabla_t w(t, x) - \mathcal{H}(t, x, -\nabla_x w(t, x)) \begin{cases} \leq 0 \\ \geq 0 \end{cases} \quad (9)$$

$\forall (t, x) \in [0, \infty) \times \mathbb{R}^n$ where, for any C^1 function $w : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$, the function $(\phi - w)(\cdot, \cdot)$ has, respectively, a local minimum and a local maximum.

This solution concept satisfies the uniqueness and nonsmoothness requirements of the generalized solution to the HJB equation, but, when endpoint state constraints are present, a characterization of an extended valued, lower semicontinuous solution is needed. So, we need the concept of generalized gradient, more specifically the proximal sub-gradient, which is used here.

The *proximal sub-gradient* of a function Φ at (t, x) , denoted by $\partial_P \Phi(t, x)$, is the set of all vectors $(\alpha, \xi) \in \mathbb{R}^{1+n}$ such that there exists $\sigma > 0$ and a neighborhood U of (t, x) with

$$\Phi(\tau, y) \geq \Phi(t, x) + \alpha(\tau - t) + \langle \xi, y - x \rangle - \sigma(\|\tau - t\|^2 + \|y - x\|^2), \quad (10)$$

for all $(\tau, y) \in U$. Analogously, the *proximal super-gradient* of a function Φ at (t, x) , denoted by $\partial^P \Phi(t, x)$, is the set of all vectors $(\alpha, \xi) \in \mathbb{R}^{1+n}$, such that there exists $\sigma > 0$ and a neighborhood U of (t, x) with

$$\Phi(\tau, y) \leq \Phi(t, x) + \alpha(\tau - t) + \langle \xi, y - x \rangle - \sigma(\|\tau - t\|^2 + \|y - x\|^2), \quad (11)$$

for all $(\tau, y) \in U$. Alternatively, the proximal super-gradient can be defined by $\partial^P \Phi(t, x) = -\partial_P(-\Phi)(t, x)$.

Now, we are equipped to present the concept of proximal solution to the HJB equation.

Definition 1 A lower semicontinuous function $v : [\tau, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R} \cup \{+\infty\}$ is a proximal solution to the HJB equation if $\forall (t, x) \in [\tau, \infty) \times \mathbb{R}^n$, such that $\partial^P v(t, x) \neq \emptyset$,

$$\eta_0 - \mathcal{H}(t, x, -\eta) = 0, \quad \forall (\eta_0, \eta) \in \partial^P v(t, x). \quad (12)$$

There are well known results in the literature providing a characterization of the value function, $V : R \times \mathbb{R}^n \rightarrow R$, for an optimal control problem (for our problem in this article $V(\tau, z) := \inf\{P_\infty(\tau, z)\}$) as a generalized lower semicontinuous solution to the HJB equation (see for example Theorem 12.3.7 in [18]). For the infinite time horizon, such a result was derived in [2].

Clearly, invariance type results provide more detailed information on optimal control processes than this characterization of the value function and thus we proceed with the definition and properties of verification functions in the next section.

4 Local Verification Functions

Next we provide a result, which is standard in Dynamic Programming, for the conventional optimal control problem. In fact, we extend the concept of local verification function for this new problem formulation and provide weak conditions under which

the existence of a verification function for a reference process $(\bar{x}, \bar{\xi}, \bar{u})$ is a necessary and sufficient condition for it to be optimal.

Let \bar{x} be an admissible arc of problem $P_\infty(\tau, z)$. Introduce the tube $T(\bar{x}, \epsilon)$ about \bar{x} :

$$T(\bar{x}, \epsilon) := \{(t, x) \in [\tau, \infty) \times \mathbb{R}^n : \|x - \bar{x}(t)\| \leq \epsilon\} \quad (13)$$

Definition 2 A function $\phi : T(\bar{x}, \epsilon) \rightarrow \mathbb{R} \cup +\infty$ is a lower semicontinuous local verification function for $(\bar{x}, \bar{\xi}, \bar{u})$ if ϕ is lower semicontinuous and the following conditions are satisfied.

1. $\forall (t, x) \in \text{int } T(\bar{x}, \epsilon)$ such that $\partial^P \phi(t, x) \neq \emptyset$,

$$\eta^0 + \min_{u \in \Omega} \{\eta \cdot f(x, u) + e^{-\delta t} f_0(x, u)\} \geq 0,$$

$$\forall (\eta^0, \eta) \in \partial^P \phi(t, x).$$

2. $\liminf_{t \rightarrow \infty} \phi(t, \xi) \leq g(\xi) + \int_\tau^\infty e^{-\delta t} f_0(x(t), u(t)) dt$, $\forall \xi \in S$ for any admissible control process (x, u) .

3. $\liminf_{t \uparrow \infty, \xi' \rightarrow \xi} \phi(t, \xi') = \liminf_{t \uparrow \infty} \phi(t, \xi)$ for all $\xi \in S \cap [\bar{\xi} + \epsilon B]$.

4. $\phi(\tau, z) = g(\bar{\xi}) + \int_\tau^\infty e^{-\delta t} f_0(\bar{x}(t), \bar{u}(t)) dt$.

We have the following necessary and sufficient conditions of optimality.

Theorem 1 Let $(\bar{x}, \bar{\xi}, \bar{u})$ be an admissible process of problem $P_\infty(\tau, z)$. Assume that the basic hypotheses (H1) – (H5) hold. We have the following.

1. If there exists a lower semicontinuous local verification function for $(\bar{x}, \bar{\xi}, \bar{u})$, then this control process is a strong local minimizer for $P_\infty(\tau, z)$.
2. Conversely, if $(\bar{x}, \bar{\xi}, \bar{u})$ is a strong local minimizer of $P_\infty(\tau, z)$ and $|g(\xi)| + |\int_\tau^\infty f_0(x(t), u(t)) dt|$ is bounded for all admissible processes (x, ξ, \bar{u}) , then there exists a lower semicontinuous local verification function for $(\bar{x}, \bar{\xi}, \bar{u})$.

The proof is a slight modification of a similar result for finite time interval problems in [18].

Take $T > 0$ large with $T > \tau + 1$ and consider the approximate problem of $P_T(\tau, z)$ of $(P_\infty(\tau, z))$.

$$\text{Min} \quad J(x, u) \quad (14)$$

subject to (1), (2), and

$$x(T) \in S. \quad (15)$$

Here B is the unit ball of \mathbb{R}^n and $J(x, u)$ is given by

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$$g(x(T)) + \int_{\tau}^T e^{-\delta t} f_0(x(t), u(t)) dt + \int_{T-1}^T e^{\gamma s} \|x(s) - x(T)\| ds.$$

The logic behind this approximating problem is that when $T \rightarrow \infty$ the last term in the cost function is hopefully forced to go to zero and we can show that there is subsequence $x(T_k)$ that converges to some point $\xi \in S$.

Let us now define verification function for a process (\bar{x}, \bar{u}) of problem $P_T(\tau, z)$.

Definition 3 $\phi : T(\bar{x}, \delta) \rightarrow \mathbb{R} \cup \{\infty\}$ is a lower semicontinuous local verification function for \bar{x} with parameter $\delta > 0$ if ϕ is lower semicontinuous and satisfies:

$$a) \quad \forall (t, x) \in \text{int}T(\bar{x}, \delta) \text{ such that } \partial^P \phi(t, x) \neq \emptyset,$$

$$\eta^0 + \min_{u \in \Omega} \{ \langle \eta, f(x, u) \rangle + f_0(x, u) \} \geq 0 \quad (16)$$

$$\forall (\eta^0, \eta) \in \partial^P \phi(t, x).$$

$$b) \quad \phi(T, \zeta) \leq g(z), \quad \forall \zeta \in S.$$

$$c) \quad \lim_{t' \uparrow T, x' \rightarrow \zeta} \inf \phi(t', x') = \phi(T, \zeta), \quad \forall \zeta \in S \cap [\bar{x}(T) + \delta B].$$

$$d)$$

$$\phi(\tau, z) = g(\bar{x}(T)) + \int_{\tau}^T e^{-\delta t} f_0(\bar{x}(t), \bar{u}(t)) dt + \int_{T-1}^T e^{\gamma s} \|\bar{x}(s) - \bar{x}(T)\| ds.$$

We have the following result which can be found in [18].

Theorem 2 Let (\bar{x}, \bar{u}) be an admissible process of problem $P_T(\tau, z)$. Assume that the basic hypotheses (H1) – (H5) hold. We have the following:

1. If there exists a lower semicontinuous local verification function for (\bar{x}, \bar{u}) , then this control process is a strong local minimizer for $P_T(\tau, z)$.

2. Conversely, if (\bar{x}, \bar{u}) is a strong local minimizer of $P_T(\tau, z)$ and $|g(\bar{x}(T))| + \left| \int_{\tau}^T f_0(\bar{x}(t), \bar{u}(t)) dt + \int_{T-1}^T e^{\gamma s} \|\bar{x}(s) - \bar{x}(T)\| ds \right|$ is bounded for all admissible processes (\bar{x}, \bar{u}) , then there exists a lower semicontinuous local verification function for (\bar{x}, \bar{u}) .

The algorithm for feedback control synthesis for problem $P_{\infty}(\tau, z)$ is presented and discussed in this section.

We draw the attention for the fact that the algorithm constructed here yields an approximation to the minimum, i.e., the true optimal solution.

In this construction we use a standard procedure [18] modified in order to force the state to reach the target set S . Moreover, once S is attained, the state is forced to remain there until the optimal point $\xi \in S$ is reached in finite time or asymptotically approached.

A partition $\pi = \{t_k\}$ of $[\tau, \infty)$ is a countably, strictly increasing sequence t_k such that $t_i > t_j$, whenever $i > j$, $t_k \rightarrow \infty$ as $k \rightarrow \infty$. The diameter of π , denoted by h_{π} , is defined by $\sup_{k \geq 0} \{\Delta_k\}$, where $\Delta_k = t_{k+1} - t_k$. Let us assume that $\tau = 0$.

Let ϕ be a given local verification function as defined in the previous section computed according to the definition.

Let $x \in \mathbb{R}^n$ be a given state. Define

$$U(x) := \{u \in \Omega, \langle N_S^P(p_S(x)), f(x, u) \rangle \leq 0\}$$

where $p_S(x)$ is the proximal point of x at S .

Let us start with $x(0) = x_0$. Then, an approximating optimal control process is constructed recursively by computing a piecewise constant control function given, for each $k = 0, 1, \dots$ by

$$\bar{u}_k^{\pi} \in \arg \max_{u \in U(x^{\pi}(t_k^{\pi}))} \left\{ \phi(t_k^{\pi}, x^{\pi}(t_k^{\pi}) + \Delta_k f(x^{\pi}(t_k^{\pi}), u)) + \Delta_k f^0(x^{\pi}(t_k^{\pi}), u) \right\}$$

and the corresponding trajectory is obtained by integrating the dynamics differential equation with the obtained control and the boundary condition given by the last value of the state variable in the previous time subinterval of the partition. Namely, $x^{\pi}(t)$ is defined on $[t_k^{\pi}, t_{k+1}^{\pi})$ as the solution of

$$\dot{x}(t) = f(t, x(t), \bar{u}_k^{\pi}) \quad \text{a.e. } t \in (t_k^{\pi}, t_{k+1}^{\pi}],$$

with initial value $x(t_k^{\pi})$ given by the value of the state variable in the previous interval.

We have the following main result of this work.

Theorem 3 Assume that hypotheses (H1)-(H5) hold true. Let ϕ be a lower semicontinuous solution to the Hamilton-Jacobi-Bellman equation. Take (x^{π}, u^{π}) , the control process obtained by the recursive procedure described above. Then, x^{π} has a cluster point with respect to the topology of uniform convergence on compact intervals, and, associated with such a point $x(\cdot)$, there is a pair, control $u(\cdot)$ and limit point ξ , such that $(x(\cdot), \xi, u(\cdot))$ is an optimal process of $P_{\infty}(0, x_0)$.

We just outline the proof here. Given $N \in \mathcal{N}$, there exist a partition $\pi(N)$, and a corresponding process (x^π, u^π) constructed by the procedure described above, such that (x^π, u^π) restricted to the time interval $[0, T_N]$ is admissible for the slightly perturbed problem $P_{T_N}(0, x_0)$, defined by:

$$\text{Min} \quad J(x, u) \quad (17)$$

subject to (1), (2), and

$$x(T_N) \in S + \frac{1}{N}B. \quad (18)$$

Let us denote this problem by P_N . Our aim is to extract the limit as $N \rightarrow \infty$ to recover the original problem. So, we assume that the sequence T_N satisfies $T_N \rightarrow \infty$ as $N \rightarrow \infty$.

Under assumptions (H1) – (H5), it is easy to show that $\{x^\pi(\cdot)\}$ has a cluster point, $\bar{x}^N(\cdot)$, on $[0, T_N]$. Here, we are using the topology of uniform convergence on $[0, T_N]$. It is also possible to show that, by using Filippov's selection theorem (see [18], for example) that there exists \bar{u}^N such that $(\bar{x}^N(\cdot), \bar{u}^N(\cdot))$ is an optimal process for P_N .

Since $\bar{x}^N(T_N) \in S + (1/N)B$ and S is compact, there exists a subsequence of $\bar{x}^N(T_N)$ converging to some point $\xi \in S$. We denote it by $\{\bar{x}_k^N\}$ and consider the corresponding processes $(\bar{x}_k^N(\cdot), \bar{u}_k^N(\cdot))$ on the respective intervals $[0, T_k]$, $k = 1, 2, \dots$. T_k is a subsequence of T_N .

Now, we show how to obtain the optimal control process for the original problem. Restrict $(\bar{x}_k^N(\cdot), \bar{u}_k^N(\cdot))$ to $[0, T_1]$. Again it is possible to show that $\bar{x}_k^N(\cdot)$ has a cluster point, $x^1(\cdot)$, in the uniform convergence topology on $[0, T_k]$ and there exists a corresponding control function $u^1(\cdot)$. Now, by considering this subsequence $(\bar{x}_k^N(\cdot), \bar{u}_k^N(\cdot))$ restricted to $[0, T_2]$ for $k = 2, 3, \dots$ and repeating the same argument as before, we can find a process $(x^2(\cdot), u^2(\cdot))$ satisfying (1) and (2) restricted to $[0, T_2]$, in which $(x^2(\cdot))$ is a cluster point of $(\bar{x}_k^N(\cdot))$.

Continuing this process, for all $k \in \mathcal{N}$, we can show $(x^k(\cdot), u^k(\cdot))$ satisfying (1) and (2) restricted to $[0, T_k]$, in which $(x^k(\cdot))$ is a cluster point of $(\bar{x}_k^N(\cdot))$.

For each $T > 0$, there exists $k \in \mathcal{N}$ such that $T \in [T_{k-1}, T_k]$.

Define $(\bar{x}(t), \bar{u}(t)) : [0, \infty) \rightarrow \mathbb{R}^n \times \mathbb{R}^m$ by

$$(\bar{x}(t), \bar{u}(t)) \equiv (x^k(t), u^k(t)) \quad t \in [0, T]. \quad (19)$$

Lemma 4 *We have the following.*

1. *The function given by (19) is well defined.*
2. *$((\bar{x}(t), \bar{\xi}, \bar{u}(t)))$ is an optimal process for $P_\infty(0, x_0)$.*

$(\bar{x}(t), \bar{u}(t))$ is well defined by construction. The second assertion follows from the existence of ϕ and from Theorem 1.

6 CONCLUSIONS

We propose and study an infinite time horizon control optimization problem in which a given objective functional is optimized by choosing control strategies which ensure the stabilization of the dynamic control systems within a given target set for the case in which the system is set invariant. Therefore, the trajectory associated the optimal control process converges asymptotically to an optimal equilibrium within a given target set. We provided a dynamic programming based algorithm which yields an control process defined in a feedback form that approximates the optimal process whose state trajectory state approximates exponentially the optimal equilibrium point.

The method proposed here is modification of previous construct in [12] for a simpler problem. In this article, the model is finite time interval and there are no target set or set constraints.

7 ACKNOWLEDGMENTS

The authors gratefully acknowledge the mobility support provided by the grant under the bilateral cooperation agreement Fapesp-GRICES, the grant from CNPq, and the support from the FCT funded research project POSC/EEA-SRI/61831/2004 “Controlo Óptimo com Restrições e Aplicações”.

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